CHAPTER 11

Internationalism and Women Mathematicians at the University of Göttingen

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The focus of this chapter is the beginning of (official) women's study of mathematics in Germany, which started at the little University of Göttingen, then situated in the largest German federal state, Prussia. It will demonstrate the decisive role that the mathematician Felix Klein (1849–1925) played as one of the foremost promoters of women studying at the university level as well as the supporting role played by David Hilbert (1862–1943) in these developments. The discussion will be embedded in an international framework.

Non-German Women Paving the Way in Germany

Although women were not legally permitted to enrol in German universities during the nineteenth century, the first woman to earn a doctoral degree in mathematics nevertheless did so at the University of Göttingen in 1874: the Russian Sofia Kovalevskaya (1850–1891). The life and work of Kovalevskaya—and the circumstances of her doctorate *in absentia*—have received sufficient scholarly attention. Kovalevskaya's Ph.D. examination records were first published in Tollmien (1997; see also Tobies 2001; Kaufholz-Soldat and Oswald 2020). It should be stressed that Kovalevskaya's career, until she became a full professor in Stockholm, had been assisted by mathematicians

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from Sweden, Germany, France, and Italy (Coen 2012, pp. 477, 509–15). Felix Klein, too, then a young professor at the University of Erlangen (in the German federal state of Bavaria), immediately recognized the significance of Kovalevskaya's thesis, '*Zur Theorie der partiellen Differentialgleichungen*' (1875) and praised it in a letter to his main collaborator at that time, the Norwegian Sophus Lie (1842–1899) (Tobies 2019, p. 368; Tobies 2021, p. 411).

Kovalevskaya's career, however, was an exception, and it was not until 1895 that the next women mathematicians completed their doctorates at German universities. Marie Gernet (1865–1924) would become the first German to do so, in 1895 at the University of Heidelberg (in the German federal state of Baden) and under the direction of Leo Königsberger (1837–1921), with whom Kovalevskaya had begun her studies in 1869. Marie Gernet became a teacher at the first German (private) secondary school for girls in Karlsruhe (Baden) where it was possible to take the *Abitur*, the examinations required for entrance to German universities. The next German woman to complete a doctorate in mathematics at a German university (Erlangen) was the famous Emmy Noether (1882–1935), who is discussed later in this chapter, but this was not until 1908.

In the meantime, women's study was further developed in other countries. Felix Klein, a corresponding member of the British Association for the Advancement of Science since 1873, a foreign member of the London Mathematical Society since 1875, and a participant in other foreign institutions, saw and recognized these new trends and supported mathematically gifted students regardless of their sex, religion and nationality. From the early stages of his career, he co-operated with a number of international colleagues in the field of geometry who likewise supported women mathematicians. These included Gaston Darboux (1842–1917) in France, Luigi Cremona (1830–1903) in Italy, Arthur Cayley (1821–1895) in the United Kingdom, Hieronymus G. Zeuthen (1839–1920) in Denmark and James Joseph Sylvester (1814–1897) in the United Kingdom and in the United States as well.[1]

From an international perspective, between the year 1874 (the year of Kovalevskaya's doctorate) and 1895, five other female mathematicians received a doctoral degree. The next was the Russian Elizaveta Fedorovna Litvinova (1845–1919)—a friend of Kovalevskaya's—at the University of Bern (Switzerland) in 1878, since Swiss universities officially allowed women's studies earlier than German universities. Incidentally, a German female mathematician also received her doctoral degree from the University of Bern, in 1907, when not all German universities had opened their doors to women. The latter, Annie Reineck (1880–1978), is notable because she was able to continue her occupation as a teacher in Switzerland after she married, whereas in Germany she would have lost her position because of the so-called *Beamtinnenzölibat* which demanded female teacher celebacy. With the support of her husband, Reineck-Leuch took on a leading position in the *Schweizerischen Verband für Frauenstimmrecht*.[2]

Back to the five women, mentioned above, who earned degrees between 1874 and 1895: Litvinova was followed by the Englishwoman Charlotte Angas Scott (1858–1931), from the University of London, in 1885. She had previously studied at the University of Cambridge. The latter university did not allow women to pursue a doctoral degree or obtain a formal research position, although a remarkable number of women did earn undergraduate degrees in mathematics there, first from Girton College in 1873 and then from Newnham College in 1875.[3] On these grounds, some of them tried their luck abroad. Recommended by the mathematician Arthur Cayley, Charlotte Angas Scott became chair of the mathematics department at the newly founded Bryn Mawr College for women in Pennsylvania (United States), where she supervised female doctoral students and maintained contact with Göttingen. For the same reason, Grace E. Chisholm (1868–1944), who graduated from Girton in 1892, would also go to Göttingen, which means to Felix Klein.

Klein, who was invited to succeed Sylvester at Johns Hopkins University, Baltimore, in 1883, knew of the developments taking place in North America where the next three women would receive a doctoral degree in mathematics. Winifred Harington Edgerton (1862–1951) was the first to do so, with a thesis submitted at Columbia University in 1886. She was followed by Ida Martha Metcalf (1857–1952) from Cornell University in 1893; and one year later, likewise from Cornell, the Canadian Annie Louise MacKinnon (1868–1940) (Fenster and Parshall 1994, p. 235; Green and LaDuke 2009). MacKinnon went on to conduct post-doctoral research in Göttingen as discussed below, after which she continued her career in the United States where there were better job opportunities. In 1895, the first female Danish mathematician received her doctoral degree in Copenhagen: Thyra Eibe (1866–1919) was supervised by one of Klein's collaborators in the field of geometry, the aforementioned Zeuthen, and by Johan Ludvig Heiberg (1854–1928), a historian of mathematics and philologist (Hoyrup 1993). In 1895, too, Felix Klein passed his first female doctoral students at the University of Göttingen: the Englishwoman Grace E. Chisholm (later Chisholm-Young), as mentioned above, and the American Mary F. Winston (1869–1959). Yet Klein had to fight for this, as will be described below.

It should be noted that it was not in Germany alone that foreigners paved the way for native women interested in science and mathematics. The situation was similar in France.[4] A well-known case is that of the Polish woman Maria Skłodowska-Curie (1867–1934), who earned a *Lizenziat* in physics (1893) and mathematics (1894) at the Sorbonne in Paris, completed her doctorate, and earned two Nobel prizes. Remarkably, the Russian mathematician Sofia Kovalevskaya had already become a member of the French Mathematical Society (*La Société Mathématique de France*) as early as 1882, before she was made a professor in Stockholm. Kovalevskaya's Finnish student Ebba Louise Nanny Lagerborg (later Cedercreutz; 1866–1950) would also complete her *Lizenziat* at the Sorbonne and become a member of the French Mathematical Society in 1890 (Rahikainen 2009).

Felix Klein, Fighting for the Right of Women to Study

In the autumn of 1893, Felix Klein made it possible for the first women to enrol at the University of Göttingen. These women were at first exclusively foreigners. In order to understand this situation, it is necessary to examine the context in closer detail.

The Humboldtian university reform in 1810 had provided a decisive impetus for mathematical research in Germany, and this attracted an increasing number of foreign students during the last third of the nineteenth century. Both women and men wished to study at the centre of scholarly activity. As early as Klein's time in Bavarian universities (Erlangen 1872–1875, and Munich 1875–1880), young men came to study with him from Scandinavia and Italy. With his move to Leipzig (Saxony) in 1880, American, Austrian (Czech, Hungarian), British, French, Italian and Russian students came to learn from him, and when, in 1886, he became a professor in Göttingen, the number of foreign students further increased. Between 1886 and 1895, ten or so Americans earned a doctoral degree under Klein's supervision. The significance of these numbers becomes clear when we learn that, throughout the 1880s and 1890s in Leipzig and Göttingen, more Americans studied mathematics under Klein and his successor at Leipzig (the Norwegian mathematician Sophus Lie) than under any professor of mathematics in the United States. It goes without saying that the subsequent development of mathematics in that country was emphatically influenced by this contact (Parshall and Rowe 1994). Moreover, two of Klein's German students, Oskar Bolza (1857–1942) and Heinrich Maschke (1853–1908), became professors of mathematics at the University of Chicago and maintained close contact with Klein in Göttingen.

While women in many other countries were already able to take qualifying examinations and even to study at university (Costas 2002), the ministerial decrees allowing women to matriculate in German states were first issued between 1900 and 1909. It should be noted that every German federal state had its own laws (Tobies 1997, pp. 18–25). However, because foreign women, like men, wanted to study where the highest standards of scholarship could be expected, for some time they attempted to gain access to German universities even while official status as students could not yet be granted to them.

Sandra L. Singer (2003) has written a detailed book about North American women at German-speaking universities during the period of 1868–1915. Before her, in a pioneering study, Margaret Rossiter underscored Felix Klein's special role in promoting American women mathematicians (1982, pp. 40–41). Singer made good use of Rositter's findings and drew upon additional archival sources. An outline of the achievements of the first women in the American mathematical community can be found in the work of Della D. Fenster and Karen H. Parshall (1994). Up until the beginning of the 1890s, Berlin was regarded as the centre of mathematics in Germany, with professors

there such as Karl Weierstrass (1815–1897), Ernst Eduard Kummer (1810–1893), and Leopold Kronecker (1823–1891). But women could not study at the University of Berlin, as Kovalevskaya experienced (she was promoted and taught privately by Weierstrass). In the 1890s, Felix Klein achieved a turning point in Göttingen, where he created the basis for an international centre for mathematics, science and technology. Yet it took some time before women could participate in university courses.

In July 1891, when Felix Klein was asked by the American Ruth Gentry (1862–1917) if she could attend his lectures and seminars, he regrettably had to turn her away.[5] Decisions of this sort fell to the conservative *Kurator* of the University of Göttingen, the legal scholar Dr. Ernst von Meier (1832–1911). A staunch opponent of women studying, he also forbade Christine Ladd-Franklin (1847–1930), who had already completed her Ph.D. requirements at Johns Hopkins, from attending Klein's lectures. She had come to Göttingen in the fall of 1891 with her husband Fabian Franklin (1853–1939), who studied under Klein. In a letter to Klein, von Meier dismissed the desire of women to study with the following curt remark: 'This is worse than social democracy, which only wants to do away with differences in property. You want to abolish the difference between the sexes.'[6]

A new situation arose in the run-up to the 1893 World's Fair in Chicago. Klein's former student Heinrich Maschke, who was by then a full professor at the University of Chicago, asked in a letter dated 8 April 1893:

> One of our students of mathematics, Miss Mary F. Winston, is applying for a scholarship, on the basis of which she intends to go to Germany next year. She has [...] talent, thinks independently, and is certainly above average. [Oskar] Bolza and I have encouraged her [...] to go to Göttingen and have just as forcefully discouraged her from going to Berlin in order to keep her away from the stiff atmosphere there. Now the question remains whether female doctoral or post-doctoral students may enrol at Göttingen or whether, if that is not the case, you think you might exert your influence to make an exception. (quoted in Tobies 1991/1992, p. 153)

In order to accomplish this and other goals, Klein sidestepped the prescribed order of command (evading the conservative university *Kurator*) and communicated directly with the influential official Friedrich Althoff (1839–1908) at the Prussian Ministry of Culture.

The ministry in Berlin was far more open to the idea of women studying than the *Kurator* in Göttingen ever was. A farsighted official, Althoff recognized the signs of the time and, on 20 May 1892, he assembled a new file with the title 'The Request of Persons of the Female Gender to Matriculate and Attend Lectures at the Royal State Universities.' The file opened with excerpts from newspapers about the ability of women to study in foreign countries. Developments abroad, in other words, influenced the decisions made at the Prussian Ministry of Culture. Thus, Klein received the following promising

message from the ministry in Berlin, on 6 July 1893, shortly before he would first travel to the United States to take part in the Mathematical Congress organized in connexion with the World's Fair:

> With respect to women studying, I can confidentially say that, as I know from Mr. Althoff, the Ministry will not hinder the matter, although it will not especially encourage such questions. Regarding their [women's] participation in lectures, this custom will also become more entrenched than limited; and if American women come to study in Germany, they will not have difficulties here. Mr. Althoff is of the opinion that, without asking, you could just arrange for your numerous female American admirers to come over. (Tobies 1991/1992, p. 154)

Klein's attitude was reinforced in the United States, where he not only confirmed Mary F. Winston's outstanding talent as a mathematician, but also witnessed women in positions that a Swiss delegate to the World's Fair described as follows:

> The Americans find nothing unusual in the fact that, for instance, a woman is the director of a national bank, as in Texas, or that women have found a place on the supervisory committees of universities or in the national department of education, and this is not to mention professorial positions, of which there are many for women [...]. Not only have universities been made available to women but also preparatory secondary education, be it in connexion with schools for boys or in parallel institutions, as in Boston [...]. America knows no difference in the practice of scientific careers between men and women [...]. At the University of Chicago, there are six female professors. (Tobies 1991/1992, p. 152)

Mary Frances Winston, an honorary fellow of mathematics at the University of Chicago, participated in the International Congress of Mathematicians in Chicago which was held 21–26 August 1893. While still in America, Klein wrote to Althoff in Berlin, saying that he would like the minister to make all the necessary preparations to ensure that Winston, 'despite the existing legal regulations of the matter,' would be admitted as a visiting student during the winter semester of 1893 to 1894. Anticipating the negative attitude of the University of Göttingen's *Kurator*, Klein added that Althoff should speak with the Minister of Culture and explain the matter in such a way that von Meier would not feel left out (Tobies 2019, p. 370; Tobies 2021, p. 413).

Having returned from Chicago to Göttingen, Klein proposed to the Ministry of Culture in Berlin that Winston, Grace Chisholm, and the American Margaret Eliza Maltby (1860–1944), be allowed to enrol in university. Despite another negative vote by the university *Kurator*, the ministry approved the application of these women within six days. The *Kurator* resigned from his position in a huff, and his successor was welcoming to women. All three women participated officially in lectures and seminars and completed their doctoral theses by 1895—Chisholm and Winston under Klein, Maltby

in the field of physical chemistry under the supervision of Walther Nernst (1864–1941).

The female students were not officially matriculated but rather possessed the status of auditors (every professor had to be asked individually for permission, which ultimately had to be granted on an individual basis by the Ministry). In the meantime, additional female students had arrived at Göttingen. Klein personally helped them to receive permission to attend courses at the University. In the autumn of 1894, for example, he wrote the following to the Prussian minister of culture Robert Bosse (1832–1901):

> Your Excellence!
>
> In addition to the two women, Miss Chrisholm and Miss Winston, who for the last year have been studying mathematical subjects at the local university and whose diligence and abilities I have repeatedly commended, there are now two new applicants, Miss MacKinnon and Miss Maddison, who are likewise requesting permission from the appropriate instructors to participate, as of next semester, in lectures on mathematics, physics, and astronomy. I have examined the qualifications of both women and am thus able to support their applications in every respect.[7]

The above-mentioned Annie L. MacKinnon gave five presentations in Klein's seminars during her time in Göttingen (1894–1895). She went on to teach mathematics at Wells College in the United States; her courses included spatial geometry, analytic geometry and differential and integral equations. Encouraged by Klein, she continued to conduct further research; in a letter to him dated 2 January 1897, for instance, she remarked:

> As promised, I am writing to you now during the Christmas vacation about the work on number theory that I told you about last summer. I find that I have both the time and desire to undertake such a study and, following your suggestion, I would like to work on it for a year in order to see what I can do with it.[8]

Ada Isabel Maddison (1869–1950) was a British woman who, like Scott and Chisholm, had studied with Arthur Cayley in Cambridge. After graduating from Girton College in 1892, she attended Bryn Mawr College in the United States, where she won a fellowship for studying abroad. After returning to Bryn Mawr, Maddison completed her Ph.D. and also translated Klein's address to the Royal Academy of Science in Göttingen on 'The Arithmetizing of Mathematics' which was published in 1896 in the *Bulletin of the American Mathematical Society*.

On 1 November 1895, the mathematician Arthur Schönflies (1853–1928), an associate professor of descriptive geometry at Göttingen, wrote the following remarks: 'We now have nine women studying mathematics, and yesterday they formed a club; they will meet once a week for coffee' (Tobies 1991/1992, p. 157). These meetings can be interpreted as the formation of

the first women's network of mathematicians. In the same year, because of growing demand for access, the Prussian ministry decided that universities only needed to provide this with a list of the female participants enrolled in courses as auditors, and so not required to take exams, rather than students proper.

Charlotte Angas Scott, who supervised doctoral theses at Bryn Mawr (the first was Ruth Gentry's in 1894, the second Ada Isabel Maddison's in 1896), arranged for further students to pursue studies at the University of Göttingen. On 19 March 1897, she wrote the following to Klein: 'I expect to send two of my best students to Göttingen next year. Both have been awarded a College Fellowship, and both are eager to study under your direction for a year, if this is agreeable to you.'[9] Thus in the fall of 1897, Emilie Norton Martin (1869–1936) and Virginia Ragsdale (1870–1945) arrived in Göttingen along with other American students. Having benefitted from their time with Klein and David Hilbert, both went on to complete their doctorates under Scott's supervision at Bryn Mawr. It should be underscored that Ragsdale contributed to Hilbert's 'sixteenth problem' (the Ragsdale conjecture). In 1900, Hilbert had presented twenty-three unsolved mathematical problems in an influential and now famous speech to the International Congress of Mathematicians in Paris. Hilbert's speech, which was delivered in German, was translated by Klein's doctoral student Mary F. Winston (married name, Newson) and published under the title 'Mathematical Problems' in the *Bulletin of the American Mathematical Society* in 1902.

Such developments continued. Male and female mathematicians, who reported enthusiastically about the stimulating atmosphere in Göttingen, encouraged additional students and colleagues to study at the university there. They arrived from North America, Russia, Denmark, Norway and elsewhere. Women from Poland and, finally, from around Germany, came there as well. Klein's first women students from the German-speaking region were Frieda Hansmann (b. 1873 in Northeim, Prussia) in the summer semester of 1895, who would later earn a doctoral degree in chemistry from the University of Bern; the second was Elsa Neumann (1872–1902), who attended Klein's lecture on technical mechanics in the summer semester of 1896 and would become, in 1899, the first woman ever to be awarded a doctoral degree from the University of Berlin (her field was physics; see Vogt 1999). Klein kept a record of the students who attended his courses. These records extend from his time as a university lecturer (the summer semester of 1871) until the year 1920, even though he had already become a professor emeritus in 1913.[10]

David Hilbert in Klein's Footsteps

David Hilbert became a full professor at the University of Göttingen in 1895, at the instigation of Klein, and was also a staunch supporter of women's right to study. Both men pursued the goal of promoting mathematical research in a broader, all-encompassing sense. On the basis of their open-minded approach

to all areas of mathematics and its application, they were able to strengthen Göttingen as an international centre for mathematical research. Klein's female doctoral students (Chisholm and Winston) had already completed their dissertations when Hilbert arrived in April of 1895. By that point, Klein had supervised almost fifty doctoral students altogether, these two women included (Klein 1923, pp. 11–13). Hilbert, previously at the University of Königsberg, did not have any doctoral students until 1898. While in Göttingen, he would go on to supervise sixty-nine in all, including six women (Hilbert 1935, pp. 431–33). As early as the summer of 1895, Klein arranged for Hilbert and himself to conduct a joint research seminar in which women also took part.

Hilbert's female doctoral students worked in his specific fields of research; one was American, three Russian and two were German women from Jewish families.[11] His first female doctoral student, Anne Lucy Bosworth (1868–1907) from the United States, was already a professor of mathematics at the Rhode Island College of Agriculture and Mechanic Arts when she came to Göttingen. She earned her degree with Hilbert in 1899 in the field of the foundations of geometry. However, Bosworth met the engineer and mathematician Theodore Moses Focke (1871–1949) in Göttingen, married him in 1901, and did not continue her academic career.

Hilbert's three Russian doctoral students belonged to a group with an excellent education in mathematics, physics, and the German language because they had each attended the famous advanced courses for women in St. Petersburg (the so-called Bestuzhev Courses), which had been established in 1878 (Borisovna 2003). These courses were taught by some of the best mathematicians from St. Petersburg, and these professors of mathematics also had close contact with Klein. As of 1895, sixteen well-educated women from here came to study at the University of Göttingen, including Helene von Bortkiewicz (1870–1939) and her friend Alexandrine von Stebnitzky (b. 1868), who were born to Polish officer families. Neither completed a doctorate in Göttingen, but they attended lectures in Klein's seminars (co-taught with Hilbert) on differential calculus and on number theory in the summer and autumn of 1895.[12] Helene von Bortkiewicz was the sister of the statistician and later professor of economics Ladislaus von Bortkiewicz (1868–1931), who had completed his doctoral degree in Göttingen under the direction of Wilhelm Lexis (1837–1914).

The first Russian woman to earn a doctoral degree under Hilbert's supervision was Ljubov Nikolaevna Zapolskaya (Sapolsky, Sapolski) (1871–1943). Having successfully completed the Bestuzhev Courses in 1894, she began her studies in Göttingen in October 1895. She attended Klein's lecture on number theory and on the theory of the top. Participating in the research seminars on number theory, Zapolskaya gave two lectures on this topic: '*Theorie der höheren Congruenzen*' on 12 February, 1896, and '*Zwei Methoden der Erzeugung eines cubischen Abelschen Körpers*' on 13 May 1896.[13] It was in this field of algebraic number theory that she wrote her doctoral thesis: '*Über*

die Theorie der relativ-cubischen Abelschen Zahlkörper' (oral doctoral examination on 29 June 1900). Zapolskaya received a post-doctoral degree from the University of Moscow in 1905 and worked as a teacher at secondary schools; from 1919 to 1923, she was a lecturer at a pedagogical institute in Ryazan, offering courses in higher mathematics, and she would go on to hold similar positions elsewhere (Makeev 2011).

Born in Simbirsk, Nadeschda Nikolaevna von Gernet (1877–1943) was educated at the women's Bestuzhev Courses in St. Petersburg, too, and she started at the University of Göttingen in the summer of 1899 where she gave a presentation on '*Die Reihe von Lagrange*' in Klein's seminar on the theory of functions.[14] She went on to complete her doctoral thesis on the calculus of variations in 1901, supervised by Hilbert. Von Gernet later became a lecturer at her alma mater in St. Petersburg, and held further positions at other academic institutions, from where she in turn sent students of her own to Göttingen. An active researcher, von Gernet published a book in 1913 on the calculus of variations. After earning her degree, she returned regularly to Göttingen in the summers before the outbreak of the First World War, participating in the meetings of the Göttingen Mathematical Society, which Klein had founded in 1892.

The third of Hilbert's Russian doctoral students, Vera Lebedeva (1880–1970), arrived in Göttingen in 1903, likewise with a diploma from the Bestuzhev Courses in St. Petersburg. Working in Hilbert's field, the theory of integral equations, she defended her thesis in 1906. She also met her future husband there, the Romanian Alexandru Myller (1879–1965), who completed his own doctorate in the same year and was also supervised by Hilbert. Both became professors at the University of Iasi in Romania—he in 1910, she in 1918. She continued to publish in German and French journals, and together they created an influential school of mathematical thought. With her appointment, in fact, Vera Myller-Lebedeva became the second female full professor of mathematics in all of Europe. It is still remarkable that they were able to hold professorships simultaneously at the same university. This was possible in Romania at the time, but not elsewhere (Abele et al. 2004, pp. 133–47; Lykknes et al. 2012).

Hilbert's female doctoral students from Jewish German families, Margarete Kahn (1880–1942) and Klara Löbenstein (1883–1968), contributed to the theory of curves topology with their doctoral dissertations, became secondary school teachers, and suffered a hard fate because of the Nazi dictatorship, although Löbenstein was able to emigrate to Argentina (König et al. 2014). Klein gradually allowed Hilbert to assume the leading role in pure mathematical research, while he himself turned his focus to applied mathematics, university administration and educational reform.

New Regulations for Women in Higher Education and Secondary Schools

Although it was not until 1908 that women could attend Prussian universities as more than mere auditors, the presence of women in university classrooms became an increasingly normal occurrence in Göttingen. In 1896, Klein offered the following response to a question about the ability of women to participate in higher education:

> I am all the more pleased to answer this question as the opinion prevailing in Germany, which is that the study of mathematics must be virtually inaccessible to women, essentially blocks all efforts directed toward the development of women's higher education. In this regard, I am not referring to extraordinary cases, which as such do not prove very much, but rather to our average experiences in Göttingen. Though this is not the place to enter more deeply into the matter, I would simply like to point out that during this semester, for instance, no fewer than six women have participated in our higher mathematics courses and *practica* and, have continually proven themselves to be equal to their male classmates in every respect. The nature of the situation is that, for the time being, these women have been exclusively foreigners: two Americans, an Englishwoman, and three Russians, but certainly no one would wish to assert that these foreign nations possess some inherent and specific talent that we are lacking, and thus that, with suitable preparation, our German women should not be able to accomplish the same thing. (Kirchhoff 1897, p. 241)

Klein's conclusion was that the infrastructure for educating girls should be enhanced in Germany. New secondary schools for girls were needed, where mathematics and the sciences would be taught. Within the framework of the educational reform movement, Klein voiced his opinion on numerous committees, in many publications and lectures, and in speeches held as a member in the Upper House (*Herrenhaus*) of the Prussian Parliament (*Landtag*); he was the only representative of the University of Göttingen in this chamber. The result was new educational and professional opportunities for women.

Klein had high regard for Thekla Freytag (1877–1932), who was the first woman in Prussia to fight for the right to pass the examination for secondary school teachers (for mathematics and scientific subjects), and he wrote about all of the obstacles that she had had to overcome to do so in 1905 (Lorey 1909; Tobies 2017). In 1908, Prussia enacted new laws, which allowed the (full-time and official) enrolment of women and the establishment of new types of secondary schools for girls, where it was possible to prepare for the end of school *Abitur* examinations. This led to even more women entering higher education. At that time, the preferred career goal of both female and male students of mathematics was to become a secondary school teacher of mathematics and other subjects (Abele et al. 2004). Klein's youngest daughter, Elisabeth (1888–1968, married name Staiger), reaped the benefits of his efforts and was able to study as an officially matriculated student (Tobies

2008). Because she became a widow in 1914, and so was not subject to the ban on married women teachers, she worked as a secondary school teacher of mathematics, physics and English, and she ultimately became the principal of a school for girls (though she was demoted when the Nazis came to power in 1933). With the right of women to matriculate and with the new opportunity of becoming secondary school teachers (and even a principal at a secondary school for girls), the number of female German students of mathematics began to increase. The files of the women (and men) who had studied mathematics (and at least two other subjects) at Prussian universities and became teachers are kept in the BBF archive in Berlin.[15] Because it was long obligatory for female civil servants to remain 'celibate,' these teachers as a rule remained unmarried or had to leave their positions if they did marry.

With the reform of mathematical and scientific education, which even during his lifetime was known as 'Klein's Reform,' the number of female students increased in general, and many new career options were made available to women and men (Tobies 2011; 2012a). A teaching programme in applied mathematics, initiated by Klein in 1898, also enabled women to become, for instance, an actuary at an insurance company or an industrial mathematician (Abele et al. 2004; Tobies and Vogt 2014). A notable example is Iris Runge (1888–1966), the eldest daughter of the Göttingen mathematician Carl Runge (1856–1927) and sister-in-law of another mathematician, Richard Courant (1888–1972); she became an impressive industrial mathematician at OSRAM and Telefunken (Tobies 2012b).

A post-doctoral degree, the *Habilitation*, was required for a professorship at German universities. Until a law was passed on 21 February 1920, this qualification was restricted to men. Cordula Tollmien (1990; 2021) has already discussed in detail how Emmy Noether needed three attempts to earn this degree, even though the Göttingen professors of mathematics supported her (see also Rowe and Koreuber 2020). Noether was denied in 1915 and 1917 because she was a woman. In 1919, Klein personally supported her *Habilitation*. Encouraged by Albert Einstein (1879–1955), Klein wrote a letter to the Prussian Ministry of Culture on 5 January 1919 in which he explained that Emmy Noether was, at that time, the most productive mathematician at the University of Göttingen (Tobies 1991/1992, p. 172). In 1919, before the official law was passed, Noether became the first woman mathematician to achieve this post-doctoral degree. In 1922, she received the title *professor*, but without pay. As of 1923, she earned income from a teaching assignment in algebra, and every semester she had to reapply for this teaching contract. Noether was never offered a full professorship in Prussia, where no woman was appointed to this role until 1945 (Noether's first full professorship was at Bryn Mawr, after her emigration). Nevertheless, it was Klein (who died in 1925) and other mathematicians in Göttingen who created the conditions that would make it possible for women to attain faculty positions at universities. Throughout all of Germany before 1945, there were only two women

who became full professors: Margarete von Wrangel (1877–1932) at the Agricultural University of Hohenheim (in Württemberg) and Mathilde Vaerting (1884–1977) at the University of Jena (Thuringia), both in 1923.

The First Female Members of the German Mathematical Society

Whereas the London Mathematical Society had been established in 1865 and the French Mathematical Society in 1874, the German Mathematical Society, *Deutsche Mathematiker-Vereinigung* (DMV) was not formed until 1890. The first women to become members of the DMV were foreigners.

The aforementioned Charlotte Angas Scott became the first female member of the DMV in 1898, when Felix Klein was its president (Toepell 1991, p. 354). She already belonged to the London Mathematical Society (Oakes et al. 2005), had joined the New York Mathematical Society in 1891, and was a founding member of the American Mathematical Society (AMS) in 1894, where she served as the first woman on its council. At that time, she and her first (female) Ph.D. student were two of nine women among an AMS membership of 250. Scott again served on the AMS Council from 1899 to 1901, and in 1905 she became the vice-president (Kenschaft 1987, p. 105). She was one of only four women—the others were the Italian Iginia Massarini (b. 1887), Vera von Schiff from St. Petersburg and Charlotte Wedell (1862–1953) from Denmark—to attend the inaugural International Congress of Mathematicians in Zürich in 1897. Felix Klein, then president of the DMV, participated and delivered an invited lecture (Curbera 2009, p. 16; Mihaljević and Roy 2019). It is noteworthy that one of these four women, Charlotte Wedell, had just studied under Klein in Göttingen and had presented a lecture in his seminar ('*Die Gauss'sche Summen*') on 13 January 1897.[16] We could call her an intellectual grandchild of Klein, given that his former doctoral student, Adolf Hurwitz (1859–1919) at the *Polytechnikum* in Zürich, had supervised Wedell's thesis: '*Applications de la théorie des fonctions elliptiques à la solution du problème des Malfatti*.' At that time, the *Polytechnikum* was not permitted to grant doctoral degrees in mathematics, and so Wedell officially completed her doctorate at the University of Lausanne in 1897.

Returning to the female members of the DMV: the next after Scott was Hilbert's above-mentioned doctoral student Nadeschda N. von Gernet, who was a member from 1901 to 1938. It should be noted that her teacher, Hilbert, served as president of the organization during the year of her appointment. In 1904, the American Helen Abbot Merrill (1864–1949) became a DMV member (Toepell 1991, p. 254). She studied at the University of Göttingen from 1901 to 1902 and earned a doctoral degree from Yale two years later (Singer 2003, p. 93). Elizabeth Buchanan Cowley (1874–1945) followed as the next female American member of the DMV in 1908 (Toepell 1991, p. 75). In that year, she received her Ph.D. from Columbia and embarked upon further studies at the Universities of Göttingen and Munich.

In 1907, she and Ida Whiteside (b. 1883) had published an article together in the journal *Astronomische Nachrichten*, for which they were awarded a prize by the German Astronomical Society.

The first Italian woman to join the DMV did so in 1905: Laura Pisati had earned her doctoral degree in Rome in 1903, taught at the Technical School 'Marianna Dionigi,' and was also—as of 26 February 1905—a member of the *Circolo Matematico di Palermo* (Toepell 1991, p. 291; Jones 2009, p. 91). Her article '*Sulla estensione del metodo di Laplace alle equazioni differenziali lineari di ordine qualunque con due variabili indipendenti*' (1905) was long a fixture in scholarly bibliographies (Ganzha et al. 2008). On account of her tragic premature death, Pisati was unable to deliver her lecture, 'Essay on a Synthetic Theory for Complex Variable Functions,' at the Fourth International Congress of Mathematicians in Rome in 1908, where she would have been the first woman to have spoken at this event. A memorial for Pisati was held during Section I of the Congress in Rome (Curbera 2009, p. 44).

Although Emmy Noether accompanied her father to the Congress in Rome, she did not give a talk there. Having just finished her doctorate under Paul Gordan (1837–1912) at the University of Erlangen, she would deliver her first lecture at the annual meeting of the DMV in Salzburg one year later. In that same year, 1909, she became the first German woman to be granted membership to the DMV (Toepell 1991, p. 276).

In subsequent years, a number of other female German mathematicians earned a doctoral degree and became members of the DMV (Tobies 2006). Additional non-German women joined as well. Here the Austrian Hilda Geiringer (1893–1973), who came to Berlin after completing her doctorate in 1917 at the University of Vienna, may be singled out. Her supervisor Wilhelm Wirtinger (1865–1945) had enjoyed Klein's inspiring seminars as a post-doctoral student at the end of the 1880s (Tobies 2019, p. 306) and recommended her to go to Germany. Geiringer herself was in contact with Klein as well, and she was made a member of the DMV in 1921.[17] She would become, in 1927, the first woman in Germany to earn a post-doctoral degree in applied mathematics at the University of Berlin (Siegmund-Schultze 1993). Olga Taussky (1906–1995), another Austrian who completed her doctorate with Klein's former student Philipp Furtwängler (1869–1940) in Vienna, became a member in 1930 when she was called upon in Göttingen to edit chapters on (algebraic) number theory in Hilbert's collected works (in doing so she would correct many errors in Hilbert's papers). Born to a Czech Jewish family, she continued her research in the United Kingdom and the United States, accompanied by her Irish husband Jack Todd (1911–2007).

Finally, mention should be given to the British mathematician Dorothy Wrinch (1894–1976), a biochemist who applied mathematical principles. She joined the DMV from 1933 to 1937 (Toepell 1991, p. 424; Senechal 2013). At that same time, however, some of the female German-speaking members of the association were forced to emigrate (including Noether, Geiringer and

Taussky) when the Nazis came to power (Siegmund-Schultze 2009; König et al. 2014).

Female Contributors to the Journal Mathematische Annalen and to the Encyklopädie Project

Klein's network was based on his scientific desire to acquaint himself with as many mathematical schools as possible. He collaborated as a peer reviewer for several journals of mathematics and he organized international exchanges of scholarship and bibliographical material. He wanted to maintain the highest international standards for his journal *Mathematische Annalen* and for the book projects under his purview. Klein was one of the chief editors of *Mathematische Annalen* from 1876 to 1924, and it is worth noting when women authors first published in this journal. The first female contributors were Winston in 1895; Scott in 1899; Lebedeva in 1907, 1909 and 1911; Emmy Noether in 1915, 1916 (four times), 1917, 1920 (two times), 1921, 1922 and 1923; and Tatyana T. Ehrenfest-Afanasyeva in 1916.

All of these women's papers were closely associated with Klein's and Hilbert's fields of research. Scott's contribution, for instance, concerns her important proof of a theorem by Max Noether (1844–1921): '*A Proof of Noether's Fundamental Theorem*' (1899). Mary F. Winston, Klein's American doctoral student, had attended his course on hypergeometric functions and in the research seminar she presented novel results in a lecture entitled '*Die gewöhnlichen Kugelfunctionen als Specialfälle der hypergeometrischen Function.*'[18] Klein accepted a short paper by her (completed October 1894) to be published in *Mathematische Annalen* (Winston 1895). Shortly thereafter, she finished her doctoral thesis: '*Über den Hermite'schen Fall der Lamè'schen Differentialgleichungen.*' It is notable that Klein cited her results in a lecture titled 'The Mathematical Theory of the Top,' which he delivered at Princeton University in October of 1896, and that Charles Hermite (1822–1901), whose equations were used by Klein, had written the following, endorsing the importance of the findings, as early as 27 January 1896: 'Le résultat concernant les formues pour le mouvement d'un corps pesant de révolution est d'une bien haute importance.'[19]

Although Klein's British doctoral student Grace Chisholm, whose life and work have been discussed in detail, did not publish an article in 'Klein's journal,' it should be pointed out that Klein encouraged her to write an elementary book on geometry and the first English textbook on set theory; the latter she wrote together with her husband, William Henry Young (1863–1942) (Mühlhausen 2020). Klein also kept this couple in mind when he hoped to produce an English version of his famous encyclopaedia, the six-volume *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer* Anwendungen (edited from 1898 to 1935). Because of the First World War, however, an English translation was never produced, though an incomplete French version did eventually appear.

At Klein's instigation, one female author did contribute to the *Encyklopädie*: the Russian mathematician Tatyana Afanasyeva (1876–1964), who had also attended the courses in St. Petersburg and from 1902 had studied in Göttingen. She took part in Klein's lectures on the encyclopaedia of mathematics (1902–1903) and, in November of 1902, she gave a presentation in Klein's seminar on the principles of mechanics entitled '*Der Begriff des unendlich Kleinen und die Ableitung der Differentialgleichungen der Bewegung in allgemeinen Koordinaten bei Lagrange*.'[20] Klein quickly recognized her talents and commissioned her and the Austrian physicist Paul Ehrenfest (1880–1933) as authors. The two had met one another in Göttingen and married on 21 February 1904. Together, they wrote the famous contribution on statistical mechanics for the fourth volume of the great *Encyklopädie*.

Emmy Noether's co-operation with Klein and Hilbert on the general theory of relativity has been discussed at length, recently by Rowe (2019). Her important paper on the topic, which includes the Noether theorems named after her, was outlined by Klein in the *Göttinger Gesellschaft der Wissenschaften* in 1918, published in the *Göttinger Nachrichten*, and ultimately submitted as her post-doctoral thesis.

That Klein was an influential role model for promoting women is clear and demonstrated by the achievements of his former students. Hilbert followed in Klein's footsteps to support mathematically-gifted women, but he was not alone; a number of Klein's students became the first mathematicians at their respective institutions to supervise female doctoral students. Examples include Adolf Hurwitz and Heinrich Burkhardt (1861–1914) in Zürich, Wilhelm Wirtinger and Philipp Furtwängler (1869–1940) in Vienna, Georg Pick (1859–1942) in Prague, Virgil Snyder (1869–1950) at Cornell University and Max Winkelmann (1879–1946) at the University of Jena. In 1919 in Prague (and under Pick's supervision), Saly Ruth Ramler (1894–1993) became the first woman there to earn a doctoral degree in mathematics (Bečvářová 2020). In Jena, Winkelmann supervised the doctoral studies of Dorothea Starke (1902–1943; married name Werner), the first woman to earn such a degree at that university; he also hired her to work as an assistant at his research institute (Bischof 2014).

Over time, the number of foreign women who earned a doctoral degree in mathematics at German universities fell as the number of German women to do so rose. Up to 1906, seven foreign women (four Russians, two Americans, and an Englishwoman) had defended a mathematical dissertation in Germany (all in Göttingen). From 1907 to 1945, only three foreign women did the same, two from Great Britain (at the Universities of Marburg and Göttingen) and one from Denmark (at the University of Freiburg). The cause of this regression was above all the First World War. Afterwards, many nations, most notably the United States and Russia, established new research centres for mathematics, so that women were then more likely to pursue doctoral research

in their home country. Nevertheless, Göttingen remained an important international centre for research up until 1933, to which point it continued to attract both male and female mathematicians from abroad.

Altogether, in Germany it was mainly non-mathematicians, especially psychologists and allied specialists, who rejected the idea that women could be capable mathematicians. In contrast, most mathematicians appreciated women's mathematical achievements, given that such results could be evaluated objectively. Yet it would be a long time before male professors would allow women to join the highest professional ranks. Of course, their marginal position in the field meant that many important results—Emmy Noether's key theorems in theoretical physics being a striking example—took a long time to achieve general recognition (Kosmann-Schwarzbach 2011). Today, female professors of mathematics are a welcome part of the profession, but they still remain in the minority (Flaake et al. 2006; Stiller 2019).

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Notes

1. Florence Nightingale (1820–1910) is regarded as Sylvester's most distinguished female British student.
2. Annie Reineck-Leuch completed her doctorate with distinction with a thesis entitled *Die Verwandtschaft zwischen Kugelfunktionen und Bessel'schen Funktionen* (Tobies 1997, pp. 41, 137).
3. See Davis's archive of female mathematicians, which includes a chronological list of graduates from the University of Cambridge (1873–1940): http://www-history.mcs.st-and.ac.uk/Davis/Indexes/xCambridge.html. Accessed on 8 August 2019.
4. This is based on a lecture given by Catherine Goldstein at the University of Würzburg in October 2015.
5. As a rare exception, Gentry had previously (in 1890 and 1891) been allowed to attend lectures in Berlin by Lazarus Fuchs (1833–1902) and Ludwig Schlesinger (1864–1933).
6. The original in German reads: '*Das ist schlimmer als die Sozialdemokratie, die nur den Unterschied des Besitzes abschaffen will. Sie wollen den Unterschied der Geschlechter abschaffen!*'. UBG (*Handschriftenabteilung der Niedersächsischen Staats- und Universitätsbibliothek Göttingen*), Cod. MS. Felix Klein, Personalia 22 L.
7. UBG, Cod. Ms. Klein, I C2: fols. 95–96.
8. UBG, Cod. MS. F. Klein 10: No. 905.
9. UBG, Cod. Ms. Klein 11: No. 947. See also Parshall 2015.
10. UBG, Cod. Ms F. Klein 7 E.
11. The doctoral files of these women are published in Tobies (1999).

12. Library of the Mathematical Institute of the University of Göttingen. Protocols (scientific notebooks) of Felix Klein's mathematics seminars 1872–1912, vol. 12, pp. 226–36, 311–22 and 330–39 Online: http://www.uni-math.gwdg.de/aufzeichnungen/klein-scans/klein/ Accessed on 13 December 2020.
13. Klein Protocols, vol. 13, pp. 8–17 and 31–39.
14. Klein Protocols, vol. 15, pp. 113–31.
15. BBF (Bibliothek für bildungsgeschichtliche Forschung), Berlin. Personal records of Prussian teachers.
16. Klein Protocols, vol. 13, pp. 124–31.
17. While writing a popular book on mathematics, which made use of Klein's educational reform ideas and his conceptual coupling of precise and approximate mathematics, Geiringer wrote two letters to Klein, dated 7 November and 3 December 1921 (UBG, Cod. MS Felix Klein 9: No. 307 and No. 308). Klein had read the manuscript of her book (Geiringer 1922, pp. 93–95) and sent comments to her.
18. Klein Protocols, vol. 12, pp. 29–32.
19. UBG, Cod. F. Klein 10: No. 632.
20. Klein Protocols, vol. 19, pp. 16–21.

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